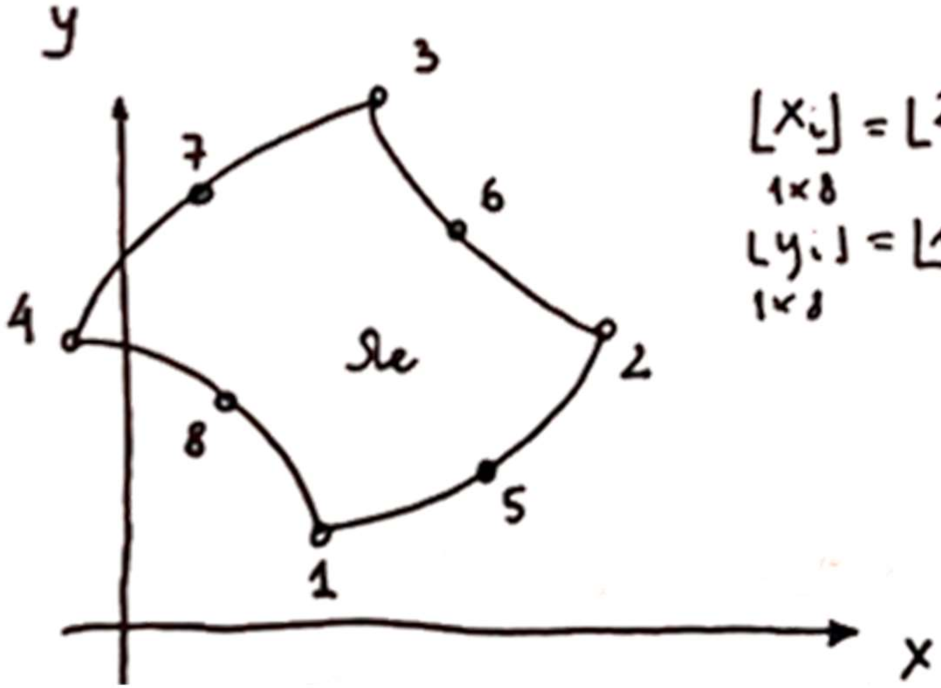
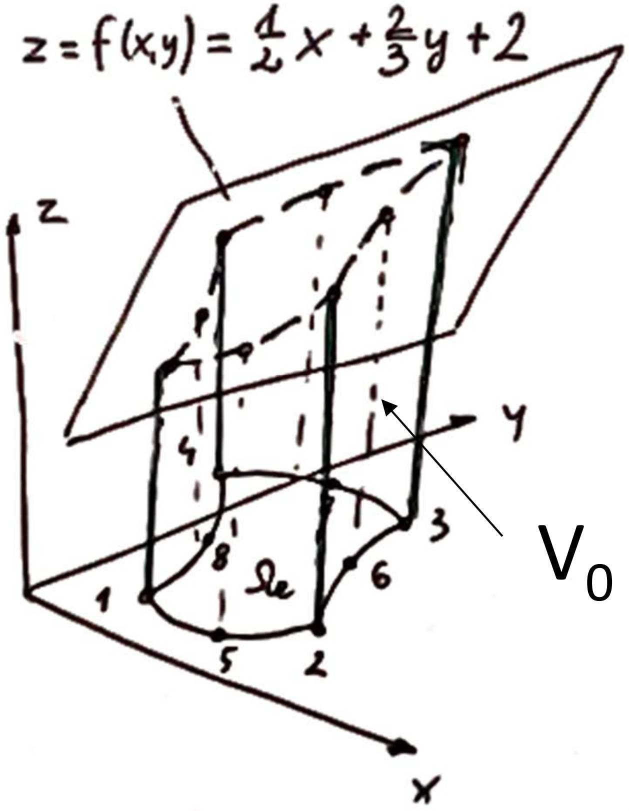
Example. QUAD-8node, numerical integration (n = 3). Find area of the finite element and volume V_0.



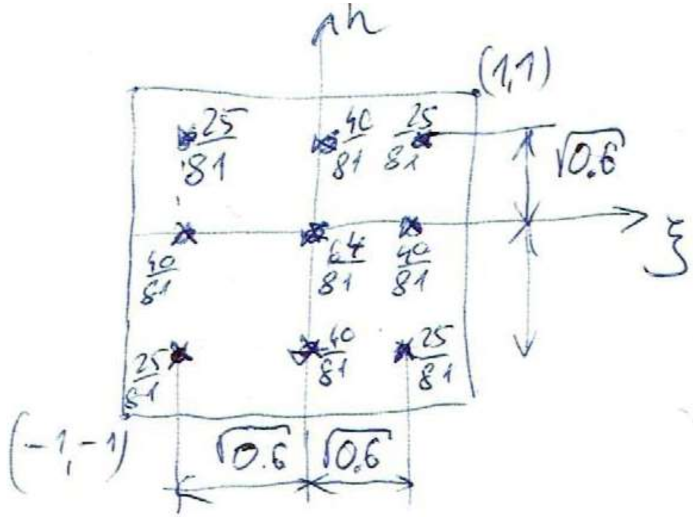
$$\underset{1\times 8}{\lfloor x_i \rfloor} = \lfloor 2, 7, 4, -1, 5, 5, 1, 1 \rfloor$$

$$\underset{1\times 8}{\lfloor y_i \rfloor} = \lfloor 1, 4, 9, 4, 2, 6, 7, 3 \rfloor$$



$$x = \lfloor N \rfloor_{1\times 8} \cdot \{x_i\}_{8\times 1}$$

$$y = \lfloor N \rfloor_{1\times 8} \cdot \{y_i\}_{8\times 1}$$



Volume:

$$V_0 = \iint_A f(x,y)\,dx\,dy =$$

$$= \int_{-1}^{1}\int_{-1}^{1} \left(\frac{1}{2}x + \frac{2}{3}y + 2\right) \det[J(\xi,\eta)]\,d\xi\,d\eta =$$

$$= \int_{-1}^{1}\int_{-1}^{1} \left(\left(\frac{1}{2} \lfloor N\rfloor\{x_i\} + \frac{2}{3}\lfloor N\rfloor\cdot\{y_i\}+2\right)\cdot\det[J(\xi,\eta)]\right)\cdot d\xi\,d\eta =$$

$$= \left| \det[J] = \frac{\partial x}{\partial\xi}\cdot\frac{\partial y}{\partial\eta} - \frac{\partial y}{\partial \xi}\cdot\frac{\partial x}{\partial\eta} = \frac{\partial\lfloor N\rfloor}{\partial\xi}\{x_i\}\cdot\frac{\partial\lfloor N\rfloor}{\partial\eta}\{y_i\} - \frac{\partial\lfloor N\rfloor}{\partial\xi}\{y_i\}\cdot\frac{\partial\lfloor N\rfloor}{\partial\eta}\{x_i\}\right| =$$

$$= \left(\frac{1}{2}\lfloor N(-\sqrt{0.6},-\sqrt{0.6})\rfloor\cdot\{x_i\} + \frac{2}{3}\lfloor N(-\sqrt{0.6},-\sqrt{0.6})\rfloor\cdot\{y_i\}+2\right)\cdot\det[J(-\sqrt{0.6},-\sqrt{0.6})]\cdot\frac{25}{81} +$$

$$+ \left(\frac{1}{2}\lfloor N(0,-\sqrt{0.6})\rfloor\cdot\{x_i\} + \frac{2}{3}\lfloor N(0,-\sqrt{0.6})\rfloor\{y_i\}+2\right)\cdot\det[J(0,-\sqrt{0.6})]\cdot\frac{40}{81} +$$

$$\ldots + (7\ \text{components}) = 220.4\ \text{mm}^3$$

Area: $A = \iint_A dx\,dy = \int_{-1}^{1}\int_{-1}^{1} \det[J(\xi,\eta)]\,d\xi\,d\eta =$

$= \det[J(-\sqrt{0.6}, \sqrt{0.6})] \cdot \frac{25}{81} + \det[J(0, -\sqrt{0.6})] \cdot \frac{40}{81} + \ldots + (7 \text{ components}) = 33.333 \text{ mm}^2$